Addition and scalar multiplication are required to satisfy these eight rules,

1. $x + y = y + x$.
2. $x + (y + z) = (x + y) + z$.
3. There is a unique "zero vector" such that $x + 0 = x$ for all x.
4. For each x there is a unique vector $-x$ such that $x + (-x) = 0$.
5. $1x = x$.
6. $(c_1 c_2)x = c_1(c_2 x)$.
7. $c(x + y) = cx + cy$.
8. $(c_1 + c_2)x = c_1 x + c_2 x$.

Suppose V = R^2, does x-y plane satisfy eight rules?

1) $x+y = y+x$

$x = (a_1, a_2)$
$y = (b_1, b_2)$

$(a_1+b_1, a_2+b_2) = (b_1+a_1, b_2+a_2)$

since $a_1+b_1 = b_1+a_1$
$x+y = y+x$ ✓

2) $x+(y+z) = (x+y)+z$

$z = (c_1, c_2)$

$(a_1, a_2) + (b_1+c_1, b_2+c_2) = (a_1+b_1, a_2+b_2) + (c_1, c_2)$

$(a_1+b_1+c_1, a_2+b_2+c_2) = (a_1+b_1+c_1, a_2+b_2+c_2)$

3) $x + 0 = x$

$0 = (0,0)$

4) $x + (-x) = 0$

$-x = (-a_1, -a_2)$

$(a_1 + (-a_1), a_2 + (-a_2)) = (0,0)$

6) $(c_1 c_2)x = c_1(c_2 x)$

$c_1 c_2 (a_1, a_2) = c_1 (c_2 a_1, c_2 a_2)$

$(c_1 c_2 a_1, c_1 c_2 a_2) = (c_1 c_2 a_1, c_1 c_2 a_2)$

Suppose $(x_1, x_2) + (y_1, y_2) = (x_1 + y_2, x_2 + y_1)$. With the usual multiplication $cx = (cx_1, cx_2)$, which of the eight conditions are not satisfied?

2) $(x + y) + z = x + (y + z)$

$(x_1 + y_2, x_2 + y_1) + (z_1, z_2) = (x_1, x_2) + (y_1 + z_2, y_2 + z_1)$

$(x_1 + y_2 + z_2, x_2 + y_1 + z_1) = (x_1 + y_2 + z_1, x_2 + y_1 + z_2)$

7) ✓ $c(x + y) = cx + cy$

$c(x_1 + y_2, x_2 + y_1) = (cx_1, cx_2) + (cy_1, cy_2)$

$(cx_1 + cy_2, cx_2 + cy_1) = (cx_1 + cy_2, cx_2 + cy_1)$

(4) $x + (-x) = 0 \qquad -x = (-x_2, -x_1)$

$(x_1, x_2) + (-x_2, -x_1) = 0$

$(x_1 - x_1, x_2 - x_2) = 0$

(1) $x + y = y + x$

$(x_1 + y_2, x_2 + y_1) \neq (y_1 + x_2, y_2 + x_1)$

(3) $(c_1 + c_2)x = c_1 x + c_2 x$

$((c_1 + c_2)x_1, (c_1 + c_2)x_2) = (c_1 x_1, c_1 x_2) + (c_2 x_1, c_2 x_2)$

$\neq (c_1 x_1 + c_2 x_2, c_1 x_2 + c_2 x_1)$

Suppose the multiplication cx is defined to produce $(cx_1, 0)$ instead of (cx_1, cx_2). With the usual addition in R², are the eight conditions satisfied?

(5) $1x = x$

$1(x_1, x_2) = (x_1, 0) \neq (x_1, x_2)$

(7) $c(x + y) = cx + cy$

$c(x_1 + y_1, x_2 + y_2) = (cx_1, 0) + (cy_1, 0)$

$(c(x_1 + y_1), 0) = (cx_1 + cy_1, 0)$ ✓

Which rule is broken if $cf(x) = f(cx)$? Keep the usual addition $f(x) + g(x)$.